Lesson 18: Least Common Multiple and Greatest Common Factor

Classwork

Opening

The *greatest common factor* of two whole numbers (not both zero) is the greatest whole number that is a factor of each number. The greatest common factor of two whole numbers a and b is denoted by GCF (a, b).

The *least common multiple* of two whole numbers is the smallest whole number greater than zero that is a multiple of each number. The least common multiple of two whole numbers a and b is denoted by LCM (a, b).

Example 1: Greatest Common Factor

Find the greatest common factor of 12 and 18.

- Listing these factor pairs in order helps ensure that no common factors are missed. Start with 1 multiplied by the number.
- Circle all factors that appear on both lists.
- Place a triangle around the greatest of these common factors.

GCF (12, 18)

12

1	12
2	6
3	4

18

1	18
2	9
3	6

Example 2: Least Common Multiple

Find the least common multiple of 12 and 18.

LCM (12, 18)

Write the first 10 multiples of 12.

12: 12, 24, (36) 48, 60, (72) 84, 96, (108) 120

Write the first 10 multiples of 18.

18: 18, (36) 54, (72) 90, (108) 126, 144, 162, 180

Circle the multiples that appear on both lists.

Put a rectangle around the least of these common multiples.

Exercises

Station 1: Factors and GCF

Choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so that the next group solves a different problem.

GCF (30, 50) 30: 1, 2, 3, 5, 6, (10), 15, 30
50: 1, 2, 5, (10) 25, 50

GCF (30, 45) 30: 1, 2, 3, 5, 6, 10, (15) 30
45: 1, 3, 5, 9, (15) 45

GCF (45, 60)
45: 1, 3, 5, 9, (15) 45
60: 1, 2, 3, 4, 5, 6, 10, 12, (15) 20, 30, 60

GCF (42, 70)
42: 1, 2, 3, 6, 7, (14), 21, 42
70: 1, 2, 5, 7, 10, (14) 35, 70

GCF (96, 144)
96: 1, 2, 3, 4, 6, 8, 12, 16, 24, 32, (48), 96
144: 1, 2, 3, 4, 6, 8, 9, 12, 16, 18, 24, 36, (48), 72, 144

Next, choose one of these problems that has not yet been solved:

a. There are 18 girls and 24 boys who want to participate in a Trivia Challenge. If each team must have the same ratio of girls and boys, what is the greatest number of teams that can enter? Find how many boys and girls each team would have.

18: 1, 2, 3, 6, 9

24: 1, 2, 3, 4, 6, 8, 12, 24

6 teams can enter the trivia challenge, each having 3 girls & 4 boys.

b. Ski Club members are preparing identical welcome kits for new skiers. The Ski Club has 60 hand-warmer packets and 48 foot-warmer packets. Find the greatest number of identical kits they can prepare using all of the hand-warmer and foot-warmer packets. How many hand-warmer packets and foot-warmer packets would each welcome kit have?

60: 1, 2, 3, 4, 5, 6, 10, (12), 15, 20, 30, 60

48: 1, 2, 3, 4, 6, 8, (12), 16, 24, 48

There can be 12 welcome kits, each having 5 hand warmers & 4 foot-warmer packets.

c. There are 435 representatives and 100 senators serving in the United States Congress. How many identical groups with the same numbers of representatives and senators could be formed from all of Congress if we want the largest groups possible? How many representatives and senators would be in each group?

435: 1, 3, 5, 87, 145, 435

100: 1, 2, 5, 10, 25, 50, 100

5 identical groups with the same # of representatives & senators can be formed, each group with 87 representatives and 20 senators.

d. Is the GCF of a pair of numbers ever equal to one of the numbers? Explain with an example.

No

e. Is the GCF of a pair of numbers ever greater than both numbers? Explain with an example.

No, Factors are by definition, less than or equal to the number. Therefore, the GCF can not be greater than both numbers.

Station 2: Multiples and LCM

Choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on the chart paper. Use your marker to cross out your choice so that the next group solves a different problem.

LCM (9,12) 9: 9, 18, 27, 36
12: 12, 24, 36 Lcm = 36

LCM (8,18) 8: 8, 16, 24, 32, 40, 48, 56, 64, 72 Lcm = 72
18: 18, 36, 54, 72

LCM (4,30) 4: 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, 44, 48, 52, 56, 60 Lcm = 60
30: 30, 60

LCM (12,30) 12: 12, 24, 36, 48, 60 Lcm = 60
30: 30, 60

LCM (20,50) 20: 20, 40, 60, 80, 100
50: 50, 100

Next, choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on this chart paper and to cross out your choice so that the next group solves a different problem.

a. Hot dogs come packed 10 in a package. Hot dog buns come packed 8 in a package. If we want one hot dog for each bun for a picnic with none left over, what is the least amount of each we need to buy? How many packages of each item would we have to buy?

10: 10, 20, 30, 40
8: 8, 16, 24, 32, 40 Lcm = 40

Four packages of hotdogs = 40 hot dogs. Five packages of buns = 40 buns.

b. Starting at 6:00 a.m., a bus stops at my street corner every 15 minutes. Also starting at 6:00 a.m., a taxi cab comes by every 12 minutes. What is the next time both a bus and a taxi are at the corner at the same time?

15: 15, 30, 45, 60
12: 12, 24, 36, 48, 60 Lcm = 60

Both the bus & taxi arrive at the corner at 7:00 am, which is 60 min after 6 am.

c. Two gears in a machine are aligned by a mark drawn from the center of one gear to the center of the other. If the first gear has 24 teeth, and the second gear has 40 teeth, how many revolutions of the first gear are needed until the marks line up again?

24: 24, 48, 72, 96, 120
40: 40, 80, 120 Lcm: 120

The first gear needs 5 revolutions (120 ÷ 24). During that time, 120 teeth pass by. The second gear revolves 3 times (120 ÷ 40).

d. Is the LCM of a pair of numbers ever equal to one of the numbers? Explain with an example.

NO

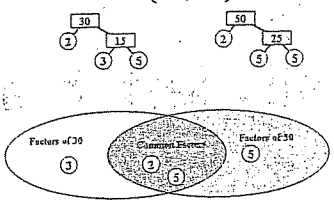
e. Is the LCM of a pair of numbers ever less than both numbers? Explain with an example.

No, Multiples are, by definition, equal to or greater than the number.

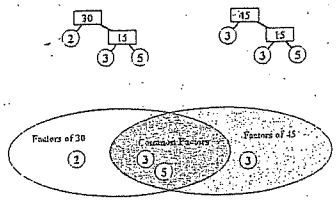
Station 3: Using Prime Factors to Determine GCF

Choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on the chart paper and to cross out your choice so that the next group solves a different problem.

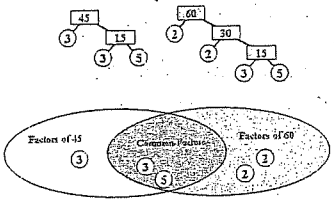
GCF (30, 50)



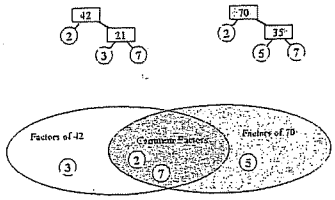
GCF (30, 45)



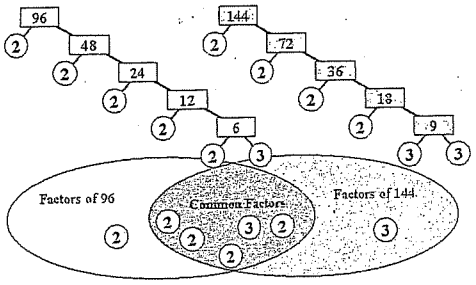
GCF (45, 60)



GCF (42, 70)



GCF (96, 144)



Next, choose one of these problems that has not yet been solved:

a. Would you rather find all the factors of a number or find all the prime factors of a number? Why?

b. Find the GCF of your original pair of numbers.

c. Is the product of your LCM and GCF less than, greater than, or equal to the product of your numbers?

d. Glenn's favorite number is very special because it reminds him of the day his daughter, Sarah, was born. The factors of this number do not repeat, and all the prime numbers are less than 12. What is Glenn's number? When was Sarah born?

$2 \cdot 3 \cdot 5 \cdot 7 \cdot 11 = 2,310$ 2/3/2010.

Station 4: Applying Factors to the Distributive Property

Choose one of these problems that has not yet been solved. Solve it together on your student page. Then, use your marker to copy your work neatly on the chart paper and to cross out your choice so that the next group solves a different problem.

Find the GCF from the two numbers, and rewrite the sum using the distributive property.

1. $12 + 18 =$ $6(2) + 6(3) = 6(2+3) = 6(5) = 30$
2. $42 + 14 =$ $7(6) + 7(2) = 7(6+2) = 7(8) = 56$
3. $36 + 27 =$ $9(4) + 9(3) = 9(4+3) = 9(7) = 63$
4. $16 + 72 =$ $8(2) + 8(9) = 8(2+9) = 8(11) = 88$
5. $44 + 33 =$ $11(4) + 11(3) = 11(4+3) = 11(7) = 77$