Teacher Notes.

Lesson 22: Writing and Evaluating Expressions—Exponents

Classwork

Example 1: Folding Paper

Exercises

1. Predict how many times you can fold a piece of paper in half.

   My prediction: ____________

2. Before any folding (zero folds), there is only one layer of paper. This is recorded in the first row of the table. Fold your paper in half. Record the number of layers of paper that result. Continue as long as possible.

Number of Folds	Number of Paper Layers That Result	Number of Paper Layers Written as a Power of 2
0	1	2^0
1	2	2^1
2	4	2^2
3	8	2^3
4	16	2^4
5	32	2^5
6	64	2^6
7	128	2^7
8	256	2^8

a. Are you able to continue folding the paper indefinitely? Why or why not?

No, the stack got too thick on one corner because it kept doubling each time.

b. How could you use a calculator to find the next number in the series?

I could multiply the number by 2 to find the number of layers after another fold.

c. What is the relationship between the number of folds and the number of layers?

As the number of folds increases by 1, the number of layers doubles.

d. How is this relationship represented in exponential form of the numerical expression?

I could use 2 as a base and the # of folds as the exponent.

e. If you fold a paper f times, write an expression to show the number of paper layers.

There would be 2^f layers of paper.

3. If the paper were to be cut instead of folded, the height of the stack would double at each successive stage, and it would be possible to continue.

a. Write an expression that describes how many layers of paper result from 16 cuts.

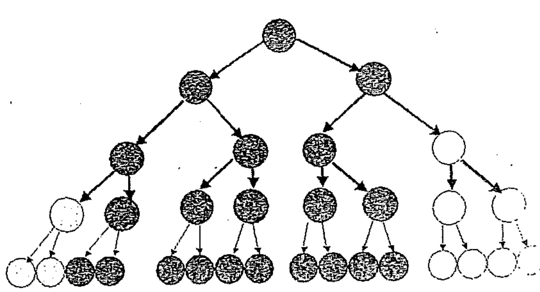
2^{16}

b. Evaluate this expression by writing it in standard form.

$2^{16} = 65,536$

Example 2: Bacterial Infection

Bacteria are microscopic single-celled organisms that reproduce in a couple of different ways, one of which is called *binary fission*. In binary fission, a bacterium increases its size until it is large enough to split into two parts that are identical. These two grow until they are both large enough to split into two individual bacteria. This continues as long as growing conditions are favorable.



a. Record the number of bacteria that result from each generation.

Generation	Number of Bacteria	Number of Bacteria Written as a Power of 2
1	2	2^1
2	4	2^2
3	8	2^3
4	16	2^4
5	32	2^5
6	64	2^6
7	128	2^7
8	256	2^8
9	512	2^9
10	1,024	2^{10}
11	2,048	2^{11}
12	4,096	2^{12}
13	8,192	2^{13}
14	16,384	2^{14}

b. How many generations would it take until there were over one million bacteria present?

20 generations will produce more than 1 million bacteria.
$2^{20} = 1,048,576$

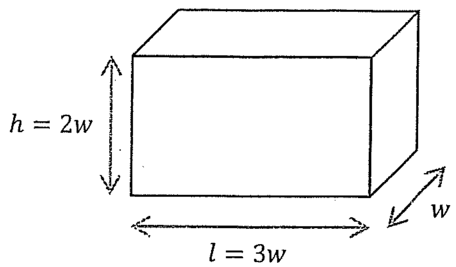
c. Under the right growing conditions, many bacteria can reproduce every 15 minutes. Under these conditions, how long would it take for one bacterium to reproduce itself into more than one million bacteria?

It would take 20 fifteen minute periods, or 5 hours.

d. Write an expression for how many bacteria would be present after g generations.

There will be 2^g bacteria present after g generations.

Example 3: Volume of a Rectangular Solid



This box has a width, w. The height of the box, h, is twice the width. The length of the box, l, is three times the width. That is, the width, height, and length of a rectangular prism are in the ratio of $1:2:3$.

For rectangular solids like this, the volume is calculated by multiplying length times width times height.

$$V = l \cdot w \cdot h$$
$$V = 3w \cdot w \cdot 2w$$
$$V = 3 \cdot 2 \cdot w \cdot w \cdot w$$
$$V = 6\,w^3$$

Follow the above example to calculate the volume of these rectangular solids, given the width, w.

Width in Centimeters (cm)	**Volume in Cubic Centimeters (cm^3)**
1	$1 \times 2 \times 3 = 6\text{cm}^3$
2	$2 \times 4 \times 6 = 48\text{cm}^3$
3	$3 \times 6 \times 9 = 162\text{ cm}^3$
4	$4 \times 8 \times 12 = 384\text{cm}^3$
w	$w \times 2w \times 3w = 6w^3\text{cm}^3$