Teacher Notes.

Lesson 8: Replacing Numbers with Letters

Classwork

Opening Exercise

$$4 + 0 = 4$$
$$4 \times 1 = 4$$
$$4 \div 1 = 4$$
$$4 \times 0 = 0$$
$$1 \div 4 = \frac{1}{4}$$

How many of these statements are true?

All of them.

How many of those statements would be true if the number 4 was replaced with the number 7 in each of the number sentences?

All of them.

Would the number sentences be true if we were to replace the number 4 with any other number?

What if we replaced the number 4 with the number 0? Would each of the number sentences be true?

No; The first 4 are true; The last one, dividing by zero is not true.

What if we replace the number 4 with a letter g? Please write all 4 expressions below, replacing each 4 with a g.

$g + 0 = g$
$g \times 1 = g$
$g \div 1 = g$
$g \times 0 = g$
$1 \div g = 1/g$

Are these all true (except for $g = 0$) when dividing?

Yes.

Example 1: Additive Identity Property of Zero

$$g + 0 = g$$

Remember a letter in a mathematical expression represents a number. Can we replace g with any number?

Yes

Choose a value for g, and replace g with that number in the equation. What do you observe?

The value of g does not change when 0 is added to g.

Repeat this process several times, each time choosing a different number for g.

Will all values of g result in a true number sentence?

Yes

Write the mathematical language for this property below:

g+0=g additive Identity Property of Zero. Any number added to zero equals itself.

Example 2: Multiplicative Identity Property of One

$$g \times 1 = g$$

Remember a letter in a mathematical expression represents a number. Can we replace g with any number?

Yes

Choose a value for g, and replace g with that number in the equation. What do you observe?

The value of g does not change when g is multiplied by 1.

Will all values of g result in a true number sentence? Experiment with different values before making your claim.

Yes.

Write the mathematical language for this property below:

g x 1 = g multiplicative identity property of one. Any number multiplied by one = itself.

Example 3: Commutative Property of Addition and Multiplication

$$3 + 4 = 4 + 3$$
$$3 \times 4 = 4 \times 3$$

Replace the 3's in these number sentences with the letter a.

$$a + 4 = 4 + a$$
$$a \times 4 = 4 \times a$$

Choose a value for a, and replace a with that number in each of the equations. What do you observe?

The result is true for all number sentences.

Will all values of a result in a true number sentence? Experiment with different values before making your claim.

Yes, any number, even zero, can be used in place of the variable a.

Now, write the equations again, this time replacing the number 4 with a variable, b.

$a+b=b+a$

$a \times b = b \times a$

Will all values of a and b result in true number sentences for the first two equations? Experiment with different values before making your claim.

Yes

Write the mathematical language for this property below:

$a+b=b+a$ Commutative Property of Addition. Order does not matter when adding,

$a \times b = b \times a$ Commutative Property of Multiplication. Order does not matter when multiplying.

Example 4

$$3+3+3+3=4\times 3$$

$$3\div 4=\frac{3}{4}$$

Replace the 3's in these number sentences with the letter a.

$a+a+a+a=4\times a$

$a\div 4=\frac{a}{4}$

Choose a value for a, and replace a with that number in each of the equations. What do you observe?

Will all values of a result in a true number sentence? Experiment with different values before making your claim.

Now, write the equations again, this time replacing the number 4 with a variable, b.

$a + a + a + a = b \times a$

$a \div b = \frac{a}{b} \quad b \neq 0$

Will all values of a and b result in true number sentences for the equations? Experiment with different values before making your claim.

In the equation $a+a+a+a = b \times a$, any value can be substituted for the variable a, but only 4 can be used for b since there are exactly 4 copies of a in the equation.

It is true for all values of a and all values of $b \neq 0$.